代数答案 Algebra 63点

2°

$(a-b)(x-a)(x-b)+(b-c)(x-b)(x-c)+(c-a)(x-c)(x-a)$

$=(a-b)(b-c)(c-a)$

Let $a=b$ then the expression;

18

$$(b-c)(x-b)(x-c)+(c-b)(x-c)(x-b)=0$$
$$(b-c)(x-b)(x-c)-(b-c)(x-c)(x-b)=0$$
$$0=0$$

Thus $(a-b)$ is a factor of the expression $(a-b)(x-a)(x-b)+\cdots$
In the same manner we can find $(b-c)$, $(c-a)$ are also the factors of $(a-b)(x-a)(x-b)+\cdots$
But $(a-b)(x-a)(x-b)+\cdots$ is the expression of third degree
Therefore there is no factor except $(a-b)(b-c)(c-a)$?

$\therefore$

$(a-b)(x-a)(x-b)+(b-c)(x-b)(x-c)+(c-a)(x-c)(x-a)$

$=(a-b)(b-c)(c-a)$.

5° $(a+b+c)^3-a^3-b^3-c^3=3(a+b)(b+c)(c+a)$

Put $a=-b$

Then $c^3+b^3-b^3-c^3 = 0$

$0 = 0$

therefore $(a+b)$ is a factor

15

in the same manner $(b+c)$, $(c+a)$ are also the factors,
but $(a+b+c)^3-a^3-b^3-c^3$ is ~~formed of the~~ the expression of third degree, therefore there is unknown some numerical multipliers; let K = numerical multiplier

and put $a=1$, $b=1$, $c=1$

then

$$27-1-1-1=K\,6$$
$$24=6K$$
$$4=K$$

therefore

$(a+b+c)^3-a^3-b^3-c^3=4(a+b)(b+c)(c+a)$.

代数答案 Algebra 63点

Algebra

1°

$a(b+c)(b^2+c^2-a^2)+b(c+a)(c^2+a^2-b^2)+c(a+b)(a^2+b^2-c^2)$

$= 2abc(a+b+c)$

Put $a=0$, then the expression;

$$bc(c^2-b^2)+bc(b^2-c^2)=0$$

i.e. $$0=0$$

Thus a is a factor of $a(b+c)(b^2+c^2-a^2)+b(c+a)(\cdots\cdots)\cdots$

In similar manner we can find that b, c are also the factors of $a(b+c)(b^2+c^2-a^2)+\cdots\cdots$.

Again put $(a+b+c)=0$ that is $a=-(b+c)$ then the expression;

~~$-(b+c)\{b^2+c^2-(b+c)^2\}+b\{c+b+$~~

$$-(b+c)\left\{b^2+c^2\ (b+c)^2\right\}+b(b-b+c)(c^2+a^2-b^2)$$

$$+c(b-b-c)(a^2+b^2-c^2)=0$$

20

代数答案　Algebra　63点

4.°

$(a+b+c)(ab+bc+ca) = (a+b)(b+c)(c+a) + abc$

~~Put $a=-b$~~

~~then~~

$\cancel{c(-b^2+bc-bc) = -b^2c}$

$\cancel{-b^2c = -b^2c}$

~~Put (a+b)~~

put $a = -(b+c)$

Then $(-b-c+b)(b+c)(c-b-c) - bc(b+c) = 0$

$bc(b+c) - bc(b+c) = 0$

$0 = 0$

$\therefore$ $(a+b+c)$ is a factor of $(a+b)(b+c)(c+a)+abc$

Again put $ab = -(bc+ca)$

Then $0 = \cancel{(a+b)c}$? $(a+b)c^2 - c(bc+ca)$

10 ⑤

$0 = \cancel{b^2(c+a)+}$ 0

$\therefore$ $ab+bc+ca$ is a factor of $(a+b)(b+c)(c+a)+abc$

Now, since $(a+b)(b+c)(c+a)+abc$ is the expression of third degree, there is no factor except $(a+b+c)$ and $(ab+bc+ca)$

$\therefore$ $(a+b+c)(ab+bc+ca) = (a+b)(b+c)(c+a)+abc$

4th grade.

63

Shiobara