代数答案 Algebra 95点

Algebra.

1° $\frac{1}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

$\therefore \quad 1 = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$

This expression being an identical we can write any number instead of x: if we put x equal to a, then,

$$1 = A(a-b)(a-c)$$

$$A = \frac{1}{(a-b)(a-c)}$$

Similarly, if we put ~~a~~ x equal ~~to~~ b and c, we can find

$$B = \frac{1}{(b-c)(b-a)} \quad \text{and} \quad C = \frac{1}{(c-a)(c-b)}$$

$$\therefore \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} = \frac{1}{x-a}\cdot\frac{1}{(a-b)(a-c)} + \frac{1}{x-b}\cdot\frac{1}{(b-c)(b-a)} + \frac{1}{x-c}\cdot\frac{1}{(c-a)(c-b)}$$

$$\therefore \frac{1}{(x-a)(x-b)(x-c)} = \frac{1}{(x-a)(a-b)(a-c)} + \frac{1}{(x-b)(b-c)(b-a)} + \frac{1}{(x-c)(c-a)(c-b)}.$$

2° $\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

$\therefore \quad x = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$

Put $x = a$, then; $\quad A = \frac{a}{(a-b)(a-c)}$

" $x = b$, then; $\quad B = \frac{b}{(b-a)(b-c)}$.

" $x = c$, " , $\quad C = \frac{c}{(c-a)(c-b)}$

$$\therefore \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} = \frac{1}{x-a}\cdot\frac{a}{(a-b)(a-c)} + \frac{1}{x-b}\cdot\frac{b}{(b-a)(b-c)} + \frac{1}{x-c}\cdot\frac{c}{(c-a)(c-b)}$$

$$\therefore \frac{x}{(x-a)(x-b)(x-c)} = \frac{a}{(x-a)(a-b)(a-c)} + \frac{b}{(x-b)(b-a)(b-c)} + \frac{c}{(x-c)(c-a)(c-b)}.$$

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3°. $$\frac{x^2}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

$$\therefore \quad x^2 = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$$

Put $x = a$, then,

$$a^2 = A(a-b)(a-c) \quad \therefore \quad A = \frac{a^2}{(a-b)(a-c)}$$

Put $x = b$ then,

$$b^2 = B(b-a)(b-c) \quad \therefore \quad B = \frac{b^2}{(b-a)(b-c)}$$

Put $x = c$ then,

$$c^2 = C(c-a)(c-b) \quad \therefore \quad C = \frac{c^2}{(c-a)(c-b)}$$

$$\therefore \frac{x^2}{(x-a)(x-b)(x-c)} = \frac{a^2}{(x-a)(a-b)(a-c)} + \frac{b^2}{(x-b)(b-a)(b-c)} + \frac{c^2}{(x-c)(c-a)(c-b)}.$$

4°. $$\frac{x^2+px+q}{(x-a)(x-b)(x-c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$$

$$\therefore \quad x^2+px+q = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$$

Put $x = a$, then $a^2+pa+q = A(a-b)(a-c) \therefore A = \dfrac{a^2+pa+q}{(a-b)(a-c)}$

Put $x = b$, then $b^2+pb+q = B(b-a)(b-c) \therefore B = \dfrac{b^2+pb+q}{(b-c)(b-a)}$

Put $x = c$, then $c^2+pc+q = C(c-a)(c-b) \therefore C = \dfrac{c^2+pc+q}{(c-a)(c-b)}$

$$\therefore \frac{x^2+px+q}{(x-a)(x-b)(x-c)} = \frac{a^2+pa+q}{(x-a)(a-b)(a-c)} + \frac{b^2+pb+q}{(x-b)(b-c)(b-a)} + \frac{c^2+pc+q}{(x-c)(c-a)(c-b)}.$$

Example 2°.

If ax^3+3bx^2+d, $bx^3+3dx+c$, have a common divisor, then $(4bd-ac)^3+27(ad^2+b^2c)^2=0$

$$b(ax^3+3bx^2+d) = abx^3+3b^2x^2+bd$$
$$a(bx^3+3dx+c) = abx^3+3adx+ac$$

∴ the difference is $3b^2x^2-3adx+(bd-ac)$ (1)

$$c(ax^3+3bx^2+d) = acx^3+3bcx^2+cd$$
$$d(bx^3+3dx+c) = bdx^3+3d^2x+cd$$

∴ The difference is $(bd-ac)x^3-3bcx^2+3d^2x$

By taking away x $(bd-ac)x^2-3bcx+3d^2$ (2)

If we put $bd-ac$ equal to u

$$(1) = 3b^2x^2-3adx+u$$
$$(2) = ux^2-3bcx+3d^2$$

$$u(3b^2x^2-3adx+u) = 3b^2ux^2-3adux+u^2$$
$$3b^2(ux^2-3bcx+3d^2) = 3b^2ux^2-9b^3cx+9b^2d^2$$
$$x(9b^3c-3adu)+u^2-9b^2d^2 \quad (3)$$

$$3d^2(3b^2x^2-3adx+u) = 9b^2d^2x^2-9ad^3x+3d^2u$$
$$u(ux^2-3bcx+3d^2) = u^2x^2-3bcux+3d^2u$$
$$x^2(u^2-9b^2d^2)+9ad^3x-3bcux$$

By taking away x, $x(u^2-9b^2d^2)+9ad^3-3bcu$ (4)

but $(3) = x+\dfrac{u^2-9b^2d^2}{9b^3c-3adu}$, $(4) = x+\dfrac{9ad^3-3bcu}{u^2-9b^2d^2}$

Since (3) and (4) have a common divisor, if that common divisor is the form of $x+m$; then

$$x+m = x+\frac{u^2-9b^2d^2}{9b^3c-3adu} = x+\frac{9ad^3-3bcu}{u^2-9b^2d^2}$$

$$\therefore \quad \frac{u^2-9b^2d^2}{9b^3c-3adu} = \frac{9ad^3-3bcu}{u^2-9b^2d^2}$$

$$\therefore \quad (u^2-9b^2d^2)^2 = (9ad^3-3bcu)(9b^3c-3adu)$$

$$(u^2-9b^2d^2)^2-(9ad^3-3bcu)(9b^3c-3adu)=0$$

$$u^4-18u^2b^2d^2+81b^4d^4-(81ab^3cd^3-27b^4c^2u-27a^2d^4u+9abcdu^2)$$

If we divide out u or $bd-ac$, then

$$u\{(bd-ac)^3-18b^2d^2(bd-ac)+27(b^4c^2+a^2d^4)-9abcd(bd-ac)+81b^3d^3\}$$
$$=0$$

By taking away the common factor is and expanding the expression, we have,

$b^3d^3-3b^2d^2ac+3bda^2c^2-a^3c^3-18b^3d^3+18b^2d^2ac-9b^2d^2ac+9bda^2c^2+$
$81b^3d^3+27(b^4c^2+a^2d^4) = 0.$

i.e.

$64b^3d^3+6b^2d^2ac+12bda^2c^2-a^3c^3+27(b^4c^2+a^2d^4)=0$

i.e.

$64b^3d^3-48b^2d^2ac+12bda^2c^2-a^3c^3+27(b^4c^2+2b^2d^2ac+a^2d^4)=0$

i.e. $(4bd-ac)^3+27(b^2c+ad^2)^2=0.$ which is to be proved.

4th Grade.

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