3. We may change the form of example as follows,

$$a^3+b^3+c^3 > \frac{1}{2}\{ab(a+b)+ac(a+c)+bc(b+c)\}.$$

Now $\quad a^2+b^2 > 2ab \quad$ (Ex 1.)

Multiply both hand $(a+b)$; then,

$$a^3+ab^2+a^2b+b^3 > 2a^2b+2ab^2$$

Subtract from both hands ab^2+a^2b, then

$$a^3+b^3 > a^2b+ab^2.$$

In the same manner we have

$$b^3+c^3 > b^2c+bc^2$$

$$c^3+a^3 > c^2a+ca^2$$

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Adding these inequals together we have,

$$2(a^3+b^3+c^3) > ab(a+b)+ac(a+c)+bc(b+c)$$

Divide both hands by 2

$$a^3+b^3+c^3 > \frac{1}{2}\{ab(a+b)+ac(a+c)+bc(b+c)\}.$$

4. $\quad 9abc < a^3+b^3+c^3+a^2b+ab^2+ac^2+a^2c+bc^2+b^2c.$

Now $\quad a^2+b^2 > 2ab$

Multiplying both hands c

$$a^2c+b^2c > 2abc$$

in the same manner

$$ab^2+c^2a > 2abc$$

$$c^2b+a^2b > 2abc$$

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Adding these inequalities together,

$$a^2c+b^2c+ab^2+c^2a+c^2b+a^2b > 6abc \quad (1).$$

Again, since the arithmetical mean of any number is greater than the geometrical,

$$\therefore \quad \frac{a^3+b^3+c^3}{3} > \sqrt[3]{a^3b^3c^3}$$

$$a^3+b^3+c^3 > 3abc \quad (2).$$

Adding the two inequals (1), (2), we have

$$a^3+b^3+c^3+a^2b+ab^2+ac^2+a^2c+bc^2+b^2c > 9abc.$$

代数答案 Algebra 90点

2° $a^6+a^4b^2+a^2b^4+b^6 > a^6+2a^3b^3+b^6$

Since $a^2+b^2 > 2ab$

if we multiply both hands a^2b^2

$a^2b^2(a^2+b^2) > 2a^3b^3$

$= a^4b^2+a^2b^4 > 2a^3b^3$

Adding both hands a^6+b^6

$a^6+a^4b^2+a^2b^4+b^6 > a^6+2a^3b^3+b^6$

$> (a^3+b^3)^2$.

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1° $m^3+1 > m^2+m$

because

$m^3+1 = (m+1)(m^2-m+1)$

and $m^2+m = m(m+1)$

$\therefore$ $(m+1)(m^2-m+1) > m(m+1)$

~~divide both hands with m+1, then~~

$\therefore$ $m^2-m+1 > m$

but $m^2+1 > 2m$

if we subtract m from both hand

$m^2+1-m > m$.

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代数答案 Algebra 90点

4th Grade, C.

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K. Shibata